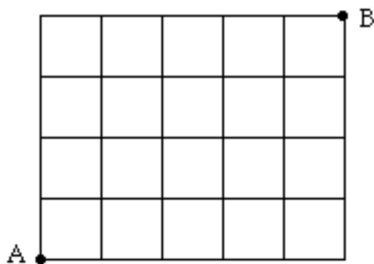


Chapter 6 Sample Problems

1. How many students must be in a class to guarantee that at least five were born on the same day of the week?
2. How many different license plates can be made if each license plate consists of three letters followed by three digits or four letters followed by two digits?
3. (a) How many functions are there from a set with three elements to a set with eight elements?
 (b) How many one-to-one functions are there from a set with three elements to a set with eight elements?
 (c) How many onto functions are there from a set with three elements to a set with eight elements?
4. What is the coefficient of x^7y^{12} in $(x + y)^{19}$ and in $(2x + 3y)^{19}$?
5. Show that $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$, whenever $n, r, k \in \mathbb{Z}^+$ with $k \leq r \leq n$ using
 (a) a combinatorial argument.
 (b) algebraic manipulation.
6. A computer is programmed to print subsets of $\{1, 2, 3, 4, 5\}$ at random. If the computer prints 40 subsets, prove that some subset must have been printed at least twice.
7. (a) How many ways are there to choose 12 cookies if there are five varieties of cookies?
 (b) How many ways are there to choose 12 cookies if there are five varieties, including chocolate chip, and at least four chocolate chip cookies must be chosen?
8. Make up a word problem in good English whose answer is $\binom{15}{4}\binom{7}{3}$.
9. How many subsets with more than two elements does a set with 100 elements have?
10. The figure below shows a 4-block by 5-block grid of streets. Find the number of ways in which you can go from point A to point B , where at each stage you can only go right or up. (You are not allowed to go left or down.) For example, one allowable route from A to B is: *Right, Right, Up, Right, Up, Up, Right, Right, Up*.



11. Here is an incorrect solution to a problem. Find the error, explain why it is not correct, and give the correct answer.
 "Problem: Find the number of ways to get two pairs of two different ranks (such as 2 jacks and 2 fives) in a 4-card hand from an ordinary deck of 52 cards."
 "Solution: There are 13 ways to get a rank (such as "kings") for the first pair and $\binom{4}{2}$ ways to get a pair of 2 that rank. Similarly, there are 12 ways to get a rank (such as "sevens") for the second pair and $\binom{4}{2}$ ways to get a pair of that rank. Therefore there are $13 \cdot \binom{4}{2} \cdot 12 \cdot \binom{4}{2}$ ways to get 2 pairs."
12. A class consists of 20 sophomores and 15 freshmen. The class needs to form a committee of size five.
- How many committees are possible?
 - How many committees are possible if the committee must have three sophomores and two freshmen?
13. Show that in a group of ten people (where any two people are either friends or enemies) there are either three mutual friends or four mutual enemies.
14. (a) Find the number of solutions to $x + y + z = 32$, where $x, y, z \in \mathbb{Z}^+$.
 (b) Answer part (a), but assume that $x \geq 7$ and $y \geq 15$.
15. Show that if five points are picked on or in the interior of a square of side length 2, then there are at least two of these points no farther than $\sqrt{2}$ apart.
16. In how many ways can five of nine people (Ann, Ben, Cal, Dot, Ed, Fran, Gail, Hal, and Ida) in a room stand in a row for a picture, if Ann and Ben are in the picture, but not standing next to each other?
17. Use the binomial theorem to:
- expand $(x^2 + \frac{1}{x})^7$.
 - prove $\sum_{i=0}^{24} \binom{50}{2i} = \sum_{i=0}^{24} \binom{50}{2i+1}$.
18. Find the number of positive integers not exceeding 1000 that are not divisible by 4, 6, or 9.